Math 55 Quiz 4 DIS 106

Name: ______________________________ 28 Feb 2022

1. Show that $(2k+3)(k-1)(k+1)$ is divisible by 3 for any positive integer k using:
   (a) Modular arithmetic and Fermat's little theorem. [5 points]
   (b) Mathematical induction [5 points]

(a)

$$\begin{aligned}(2k+3)(k-1)(k+1) &= 2k^3+3k^2-2k-3 \\ &\equiv 2k^3-2k \pmod 3 \\ &\equiv 0 \pmod 3\end{aligned}$$

where the last equivalence is due to Fermat's little theorem (since 3 is a prime).

(b) Let $P(k)$ be the proposition that $(2k+3)(k-1)(k+1)$ is divisible by 3.
$5 \cdot 0 \cdot 2 = 0$ is divisible by 3 so $P(1)$ is true.
Suppose $P(k)$ is true for some positive integer k. Then

$$\begin{aligned}(2(k+1)+3)((k+1)-1)((k+1)+1) &= 2(k+1)^3+3(k+1)^2-2(k+1)-3 \\ &= (2k^3+3k^2-2k-3)+(6k^2+6k+2+6k+3-2) \\ &= (2k+3)(k-1)(k+1)+(6k^2+12k+3)\end{aligned}$$

$(2k+3)(k-1)(k+1)$ and $6k^2+12k+3$ are divisible by 3, so $(2(k+1)+3)((k+1)-1)((k+1)+1)$ is divisible by 3. In other words, $P(k+1)$ is true.
By mathematical induction , $P(k)$ is true for any positive integer k.